comparison of the values from numerical integration and those from the series expansion. The series solution is fairly accurate for low Reynold's numbers.

HEAT TRANSFER

The present problem would be important in transpiration cooling or gaseous diffusion processes. Let the temperature of the plates at $y = 0$ and $y = d$ be T_0 and T_1, respectively. We want to calculate the heat transfer.

Substituting

$$T = T_0 + (T_1 - T_0)\theta(\eta) \tag{23}$$

into the energy equation we obtain

$$\theta'' + Pf\theta' = 0 \tag{24}$$

$$\theta(0) = 0 \qquad \theta(1) = 1 \tag{25}$$

where P is the Peclet number $(\rho g C_p V d/k)$. Equation (24) is integrated by a similar shooting method. The heat transfer rate is

$$Q(\eta) = -\frac{k(T_1 - T_0)}{d}\,\theta'(\eta) \tag{26}$$

The values $\theta'(\eta)$ for $\eta = 0$ (impermeable plate) and $\eta = 1$ (porous plate) are given in Table 2 for various Reynolds numbers and Peclet numbers. For high Peclet numbers the heat transfer on the impermeable plate is considerably higher than that of the porous plate.

NOTATION

A = constant which determines pressure distribution
C_p = specific heat at constant pressure
d = distance between plates
f = normal velocity distribution
F = normal velocity distribution
g = gravity acceleration
h = axial velocity distribution
k = thermal conductivity
L_1 = width of plates
L_2 = length of plates
P = Peclet number
Q = heat transfer per area per time
R = cross flow Reynolds number
T = temperature
T_0 = temperature at $\eta = 0$
T_1 = temperature at $\eta = 1$
u = velocity component in x direction
v = velocity component in y direction
V = velocity of injection
w = velocity component in z direction
x = cartesian coordinate
y = cartesian coordinate
z = cartesian coordinate

Greek Letters

α = proportionality constant
η = nondimensional normal distance
θ = temperature distribution
λ = $\alpha R \eta$
λ^* = nonzero root of $F'(\lambda) = 0$
ν = kinematic viscosity
ξ = $1 - \eta$
ρ = density

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Correlations for Solid Friction Factors in Vertical and Horizontal Pneumatic Conveyings

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Unlike friction factors for gases which are very well characterized with a universal measurement of turbulence, the Reynolds number, attempts to correlate solid friction factors are hampered by a lack of an unique dimensionless group. The complications stem from the fact that solid particles are usually different in size, shape, and surface roughness; that different size particles travel at different velocities and collide among one another as well as with pipe walls; that static electricity may be important, especially for small particles, but is difficult to quantify; and that the particles may not exist as a uniform suspension especially in horizontal conveying where gravitational forces tend to create a radial distribution of particle concentration.

This note presents new correlations for calculating the solid friction factors in both horizontal and vertical pneumatic conveying lines. The correlations proposed here can be used in conjunction with the modified terminal velocity equations suggested earlier (Yang, 1973a, 1973b) to calculate solid particle velocities in pneumatic conveying lines.

CORRELATIONS FOR SOLID FRICTION FACTORS

Information on the flow of fluids through beds of granular solids has been utilized to shed light on the functional dependence between different variables. Blake (1922) first obtained the correct dimensionless groups for correlating the pressure drop data in a packed bed. The suggested dimensionless groups are

$$\frac{\Delta P \cdot g_c}{2\,\rho_f\,U_0^2}\cdot\frac{d_p}{L}\cdot\frac{\epsilon^3}{(1-\epsilon)} \quad \text{and} \quad \frac{d_p\,\rho_f\,U_0}{\mu(1-\epsilon)} \tag{1}$$

The first of these groups is a modified friction factor and the second is a modified Reynolds number. Later Ergun

(1952) proposed a similar equation as shown below

$$\frac{\Delta P \cdot g_c}{2\,L} \cdot \frac{d_p}{\rho_f\,U_0^2} \cdot \frac{\epsilon^3}{(1-\epsilon)} = 75\,\frac{(1-\epsilon)}{Re} + 0.875 \tag{2}$$

Equation (2) is not only applicable for packed beds but also valid for moving beds if slip velocities $U_f - U_p$ are used in place of superficial fluid velocities, U_0 (Yoon and Kunii, 1970; Yang, 1973c). It is reasonable to assume that pneumatic conveying may behave similar to moving beds if slip velocities are used. From this preliminary assertion, data from Hariu and Molstad (1949) were plotted with $f_p\,\epsilon^3/(1-\epsilon)$ against $(1-\epsilon)/(Re)_p$. Trends were discernible; however, no unique correlation existed. When the abscissa was replaced with the dimensionless group $(1-\epsilon)(Re)_t/(Re)_p$, a unique correlation appeared as presented in Figure 1. This correlation can be expressed as

$$f_p\,\frac{\epsilon^3}{(1-\epsilon)} = 0.0206\left[\,(1-\epsilon)\,\frac{(Re)_t}{(Re)_p}\,\right]^{-0.869} \tag{3}$$

for vertical conveying

where f_p is defined in Equation (4).

$$f_p = \frac{2g_c\,D\Delta P}{\rho_{ds}\,U_p^2\,L} \tag{4}$$

The f_p used here is taken to be four times the values reported in the original paper (Hariu and Molstad, 1949). Equation (3) is good to ±30% for more than 90% of the data. For solids E and F, the deviations are more than 50%. This results from the low accuracy of U_p and U_f, as noted in the original paper. Only estimated values (by the original authors) are used here. Thus these data points are included only for comparison, but they are not included in deriving Equation (3). The voidage ϵ covered in the experimental data, ranges from 0.962 to 0.999. In view of possible error in evaluating the voidage, the correlation shown in Figure 1 is gratifying. Note that the solid friction factors do not depend on the pipe diameters in vertical pneumatic conveying.

For horizontal pneumatic conveying, the data by Hinkle (1953) were used. A different dimensionless group $(1-\epsilon)\,[(Re)_t/(Re)_p](U_f/\sqrt{gD})$ was used for the abscissa (Figure 2). The correlation can be represented by the following equation:

$$f_p\,\frac{\epsilon^3}{(1-\epsilon)} = 0.117\left[\,(1-\epsilon)\,\frac{(Re)_t}{(Re)_p}\,\frac{U_f}{\sqrt{gD}}\,\right]^{-1.15}$$

for horizontal conveying. (5)

The dependence on the pipe diameter for the horizontal pneumatic conveying is understandable. Gravitational forces, acting in the vertical direction, tend to create a particle distribution in the radial direction. This effect depends on the pipe diameter. In horizontal pneumatic conveying, solid particles may bounce between tube walls, may roll or slide at bottom of the tube, or may be suspended completely in the conveying fluid depending on particle size, shape, density, pipe diameter, fluid properties, and conveying velocity. When particles become bigger or denser as in the case of Alundum (8390μ, 1.82 g/cm^3), the particles tend to roll or slide at bottom of the tube during transport. This represents a major departure from the common pneumatic conveying and the data exhibit considerable deviation from the rest of the data as shown in Figure 2. If the data for Alundum are excluded, Equation (5) is good to ±30% for most of the data with voidage ranging from 0.994 to 0.999.

Unfortunately, most papers in the literature do not report f_p or do not contain enough information to compute f_p. Thus it is difficult to verify the extent of applicability of Equations (3) and (5). Nevertheless, this paper represents the first attempt to correlate the solid friction factors with modified Reynolds numbers for pneumatic conveying lines. The successful correlations obtained here merit further research in this direction.

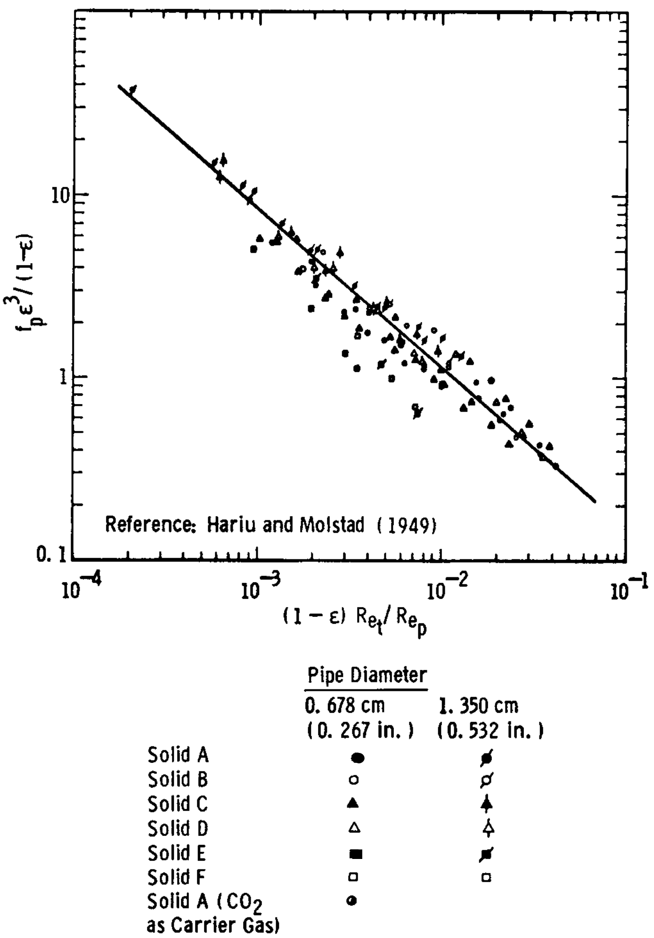

Fig. 1. Correlation of solids friction factors for vertical pneumatic conveying lines.

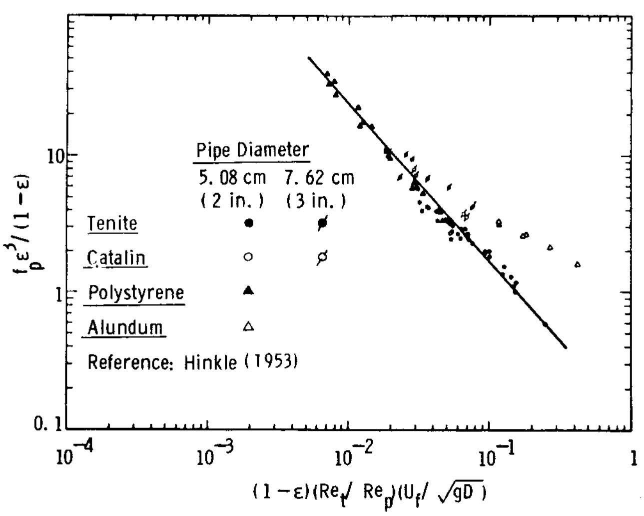

Fig. 2. Correlation of solids friction factors for horizontal pneumatic conveying lines.

CALCULATION OF SOLID PARTICLE VELOCITIES

The correlations for solid friction factors as shown in Equation (3) for vertical conveying and in Equation (5) for horizontal conveying can be combined with the modified terminal velocity equations proposed earlier (Yang, 1973a and 1973b) to calculate solid particle velocities. Trial and error solutions are required. The modified terminal velocity equations were expressed as

$$U_{tv} = \sqrt{\left(1 + \frac{f_p\, U_p^2}{2\, g_c D}\right)\left(\frac{4}{3}\right)\left[\frac{(\rho_p - \rho_f)\, d_p\, g_c}{\rho_f\, C_{DS}}\right] \epsilon^{4.7}}$$

for vertical conveying (6)

$$U_{th} = \sqrt{\left(\frac{f_p\, U_p^2}{2\, g_c D}\right)\left(\frac{4}{3}\right)\left[\frac{(\rho_p - \rho_f)\, d_p\, g_c}{\rho_f\, C_{DS}}\right] \epsilon^{4.7}}$$

for horizontal conveying. (7)

The results are presented in Figures 3 and 4. For vertical conveying, the method gives an accuracy of $\pm 30\%$ including the data for solids E and F, and, for horizontal conveying, $\pm 20\%$. A literature survey is underway to test the range of applicability for the present method.

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NOTATION

C_{DS} = drag coefficient on a single particle
d_p = mean particle diameter, m
D = inside diameter of pneumatic conveying lines, m
f_p = solid particle friction factor defined in Equation (4)
g = gravitational acceleration, m/s^2
L = length of transporting lines, m
ΔP = pressure drop in the transporting lines, Kg/m^2
Re = Reynolds number defined as $d_p\, U_0\, \rho_f/\mu$
$(Re)_p$ = Reynolds number defined as $d_p\, (U_f - U_p)\, \rho_f/\mu$
$(Re)_t$ = Reynolds number defined as $d_p\, U_t\, \rho_f/\mu$
U_0 = superficial fluid velocity based on empty column cross section, m/s
U_f = actual fluid velocity defined as U_0/ϵ, m/s
U_p = actual particle velocity, m/s
U_t = particle terminal velocity, m/s
U_{th} = modified terminal velocity for horizontal pneumatic conveying, m/s
U_{tv} = modified terminal velocity for vertical pneumatic conveying, m/s
ρ_{ds} = dispersed solids density, kg of dispersed solids/m^3
ρ_f = fluid density, kg/m^3
P_p = particle density, kg/m^3
μ = fluid viscosity, kg/m s
ϵ = voidage in transporting lines

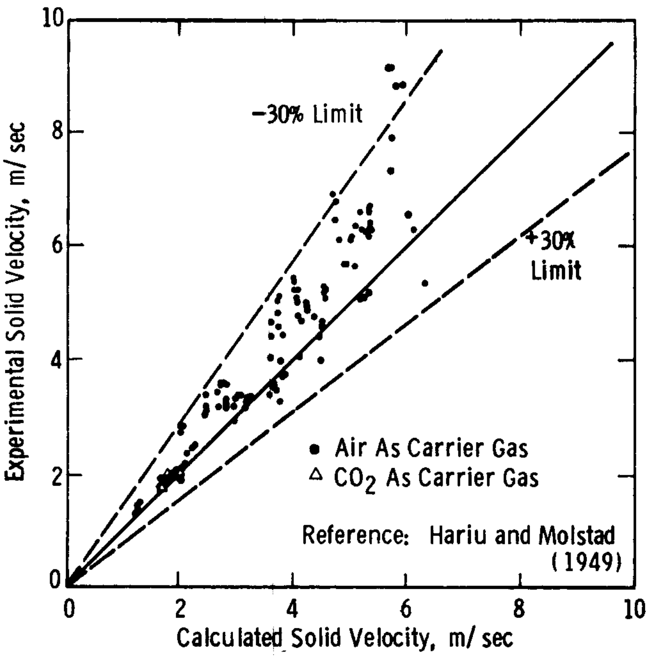

Fig. 3. Comparison between calculated and experimental values of solid particle velocity (vertical conveying).

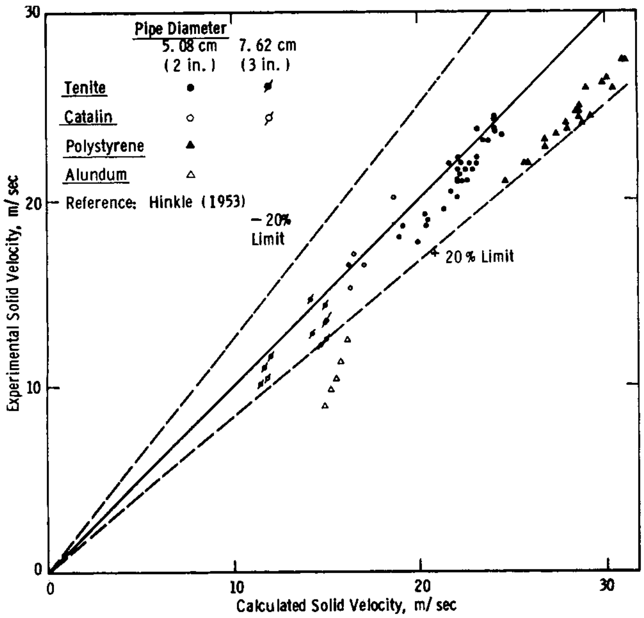

Fig. 4. Comparison between calculated and experimental values of solid particle velocity (horizontal conveying).

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